APPLICATION NOTE 2

DFF Sensor Elementary Force Measurements: Fluids, Bulk Powder and Single Granules

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Introduction

The DFF sensor works by quantifying the force exerted by the flow against the pin. Depending on the nature of the flow, the sensor either measures a continuous drag force exerted by fluid or a momentum of a particulate component of the flow. Multicomponent flows such as those realized in an industrial processing can be modeled as combination of a uniform fluid flow and a flow of particles. Connection between Lenterra's DFF sensor output and flow characteristics in such "elementary" flows may be established as follows

DFF Probe in a Uniform Fluid Flow

The force acting on the cylindrical hollow pillar in uniform flow is given by the following equation:

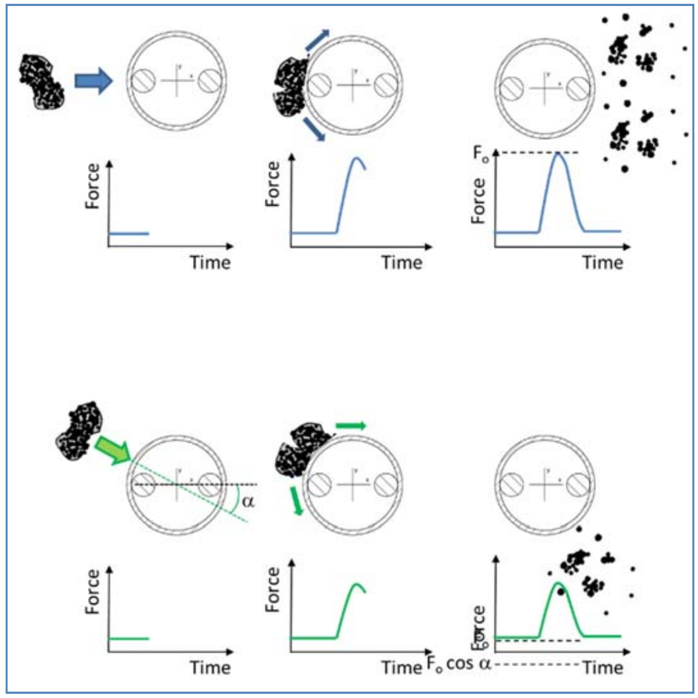
$$F = \frac{C_d}{2} A \rho v^2 , \qquad \text{Eq. 1}$$

where C_d is a drag coefficient of a cylinder, $A = 2r_o l$ is cross section of the cylinder, ρ is the fluid density, v is flow velocity. C_d is a function of the Reynolds number $\mathrm{Re}_d = \frac{2r_o v \rho}{\mu}$, where μ is the kinematic viscosity. Thus, if the flow velocity is known, the DFF sensor output is related to the density and viscosity of the fluid.

Single Particle Impact

An elementary interaction between a particle and the cylindrical probe manifests itself as a pulse in the force vs. time measurement. An impact of a single particle of mass ***m*** and velocity ***v*** colliding head-on with the DFF probe tip would cause the tip to start moving with a certain velocity ***V***.

The magnitude of the tip deviation δ_0 is determined by the relation between mass of the particle and the effective mass of the pin, $M_{eff} = \frac{1}{4\pi^2}\frac{k}{f^2}$, where k is the stiffness of the pin and f is its mechanical resonance frequency. For stainless steel pin of 40 mm length and 1 mm in diameter with 0.2 mm walls, $k \cong 360$ Nm^{-1}, $f \cong 460$ Hz, and $M_{eff} \cong 0.04$ g. Assuming a **perfectly inelastic** collision, the momentum conservation yields



Mechanistic origin of flow forces on the drag force flow (DFF) sensor. A particle colliding with the sensor in the direction of the flow generates a force that is recorded by the sensor, A: the impact is along the measurement axis of the DFF sensor (x-axis), B: the particle impact is at an angle α to the measurement axis, the detected force pulse magnitude is reduced by $\cos\alpha$.

$mv = (m + M_{eff})V$ and for $m \ll M_{eff}$ leads to $V = \frac{m}{M_{eff}} v$. The energy of the impact is transferred to the energy of the tip motion of the pin, which is related to the measured force pulse magnitude F_0 as $\frac{M_{eff}V^2}{2} = \frac{k\delta_0^2}{2} = \frac{F_0^2}{2k}$. Thus,

$$F_0 = mv\sqrt{\frac{k}{M_{eff}}} = \frac{\pi^2}{6} f v \rho d^3 \qquad \text{Eq. 2}$$

The force pulse magnitude due to interaction of a single particle is therefore proportional to the particle density $\boldsymbol{\rho}$ and third power of its diameter ***d***. Using this equation, one can estimate the smallest particle impact that will be detected by the sensor. For example, the sensor with pin parameters described above will measure force down to 0.5 mN that corresponds to the impact of the 0.7 mm diameter water droplet moving with the velocity of 2 m/s.

In the above analysis, it is assumed that the impact is along the measurement direction of the sensor. If the same impact happens at an angle α to the measurement direction (as in part B of the figure above), the sensor generates a smaller signal:

$$F_\alpha = \frac{\pi^2}{6} f v \rho d^3 cos\alpha \qquad \text{Eq. 2a}$$

Since particles in a real flow collide with the probe at various angle, impact forces registered by the DFF sensor may vary for same particle.

Probe in Powder Flow

Another limiting case occurs when a continuous one-dimensional uniform flow of identical small particles of mass m and density ρ interacts with the probe. Assuming that the particle diameter ***d*** is smaller than the diameter of the probe, the equivalent (integrated over pin length) force on the tip of the pin can be evaluated as follows (Gere & Goodno, 2013).

$$F_0 = \frac{3}{8} m v \frac{dN}{dt} = \frac{3}{8} m v \cdot nAv = \frac{\pi}{16} A \cdot \delta \rho v^2 \quad \text{Eq. 3}$$

Here, N is the total number and n is the concentration of the particles, $\boldsymbol{A}$ is the cross-sectional area of interaction, and v is the velocity of the flow. Particle number concentration is expressed as $n = \frac{\delta}{d^3}$, where $\delta < 1$ is the degree of packing. The maximum degree of packing, $\delta = 1$, is achieved when every particle is in touch with six neighboring particles (assuming cubic lattice) so that the distance between the particle centers equals to its diameter. When the distance between the particle centers is, for example, twice the particle diameter, $\delta = \frac{1}{8}$.

Assuming that the degree of packing does not change significantly during monitoring, the measured force is the function of particle density and the interaction cross section. The interaction cross section strongly depends on the degree of binding between the particles. For example, dry powders particles can be considered not bound to each other. Therefore, the interaction cross-sectional area of dry powder particles is close to the cross section of the pin (diameter times submerged length). A densely packed ($\delta = 1$) flow of particles with density ρ = 1 g/cm^3 moving perpendicularly to the probe with a velocity of 2.5 m/s, would deflect the pin equivalently to a force of 0.15 N applied to the pin tip. The lowest powder density (with $\delta = 0.1$ and v = 2 m/s) detectable by sensor with the above design parameters is therefore ρ = 0.15 g/cm^3.

Eq. 3 could also be written in terms of powder density, $\hat{\rho} = mn$, rather than particle density, ρ:

$$F_0 = \frac{3}{8} m v \cdot nAv = \frac{3}{8} A(mn)v^2 = \frac{3}{8} A\hat{\rho}v^2 \qquad \text{Eq. 4}$$

$\hat{\rho}$ can be measured, for example, by weighing the powder in a calibrated volumetric flask.

Conclusions

Rheologically complex flows can be generally characterized by a combination of a fluid, bulk powder and single particles. Due to extremely high measurement frequency, DFF sensor detects not only the bulk flow properties but also individual force impacts on the sensor probe that provides information about particle mass, size, density, and momentum. The sensor can be designed to measure flow forces in a wide range of sensitivities, and detect single particle impact as small as that formed by a 0.7 mm water droplet moving at speed of 2 m/s, or bulk powder flow of particles of density 0.15 g/cm^3.

References

Gere, J.M. & Goodno, B.J. (2013). *Mechanics of Materials*. (8th ed.). Stanford: Cengage Learning.

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